

Class 2 - Quadratic Equations

Review - Linear equations & Rational equations

1 var.

$$x^1 + 3x^1 - 10 = 9 \rightarrow \text{Lin.}$$

exp $\rightarrow 1$

$$x^{-2} + x^{3.5} + \sqrt{3x} + \frac{1}{x-4} = 1 \rightarrow \text{Not lin.}$$

Ex: $\frac{2}{x+2} - \frac{3x+1}{x^2-4} = \frac{4}{x-2}$

Restricted values: #'s denominator = zero.

$$\begin{array}{cc} x+2=0 \\ -2 \quad -2 \end{array}$$

$$\boxed{x = -2}$$

$$\begin{array}{cc} x^2-4=0 \\ +4 \quad +4 \end{array}$$

$$x^2 = 4 \rightarrow \boxed{x = \pm 2}$$

$$(-2)^2 = (-2)(-2) = 4$$

$$\begin{array}{cc} x-2=0 \\ +2 \quad +2 \end{array}$$

$$\boxed{x = 2}$$

$$\sqrt{4} = 2$$

$$x^2 = 4 \rightarrow x = \pm 2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a+b)(a-b) = a^2 - b^2$$

$$\begin{array}{cc} x^2-4 \\ \downarrow \quad \downarrow \\ a=x \quad b=2 \end{array}$$

$$x^2 - 4 = (x+2)(x-2) \rightarrow \text{GCF}$$

$$\frac{2}{\cancel{x+2}} \cancel{(x-2)(x+2)} - \left(\frac{3x+1}{\cancel{(x-2)(x+2)}} \right) \cancel{(x-2)(x+2)} = \left(\frac{4}{\cancel{x-2}} \right) \cancel{(x-2)(x+2)}$$

$$2(x-2) - (3x+1) = 4(x+2)$$

$$\underline{2x-4} - \underline{3x+1} = 4x+8$$

$$\begin{array}{r} -x-5 \\ +5 \end{array} = 4x+8 \quad +5$$

$$\begin{array}{r} -x \\ -4x \end{array} = \begin{array}{r} 4x+13 \\ -4x \end{array}$$

$$\frac{-5x}{-5} = \frac{13}{-5} \rightarrow$$

$$\boxed{x = -\frac{13}{5}}$$

Quadratic Equations

$$(2x+1)(x-4) = 2x^2 - 8x + x - 4 = 2x^2 - 7x - 4$$

AC Method

→ Factoring an expression

Ex: $ab + 10a + 2b + 20$

$$a(b+10) + 2(b+10)$$

$$(b+10)(a+2)$$

$$\hookrightarrow ab + 10a + 2b + 20 = (b+10)(a+2)$$

$$\frac{a(b+10)}{b+10} = a$$

$$\frac{2(b+10)}{b+10} = 2$$

Ex: Factor the expression $1x^2 - 7x - 30$.

↪A ↪B ↪C

AC = -30 product

B = -7 sum

$$1x^2 - 7x - 30 = (x+3)(x-10)$$

$$x^2 + 3x - 10x - 30$$

$$x(x+3) - 10(x+3)$$

$$(x+3)(x-10)$$

$$\frac{x(x+3)}{x+3} = x \quad \& \quad \frac{-10(x+3)}{x+3} = -10$$

Product

Sum

-1 & 30 →

-29

30 & -1 →

29

2 & -15 →

-13

-2 & 15 →

13

-6 & 5 →

-1

6 & -5 →

+1

-3 & 10 →

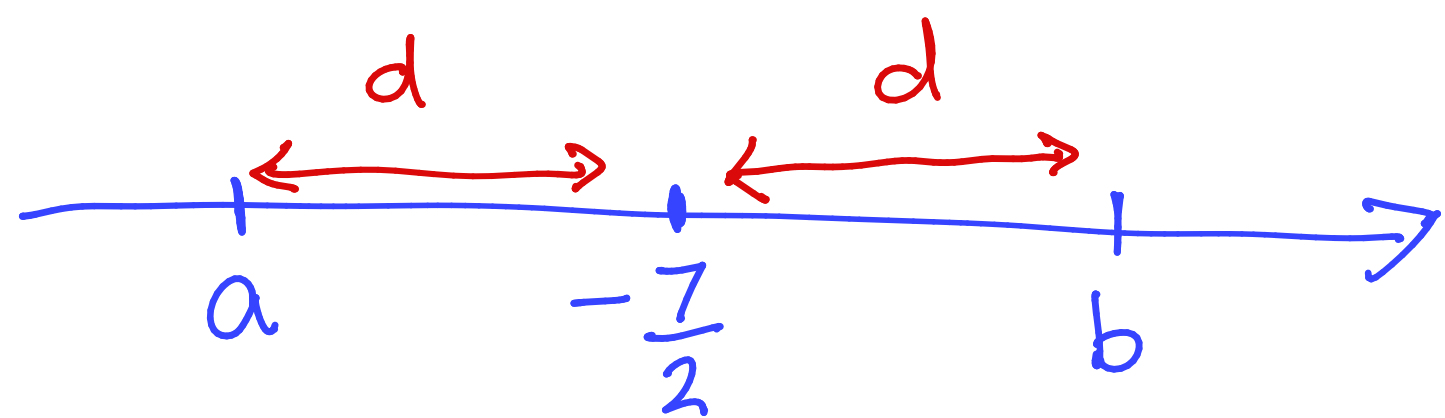
7

3 & -10 →

-7

↑
(x+3)(x-10)

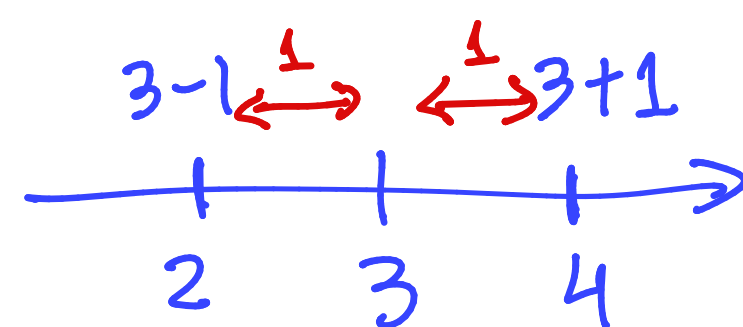
$$x^2 - 7x - 30 = (x + a)(x + b)$$



$$\frac{a+b}{2} = \frac{-7}{2} \rightarrow \boxed{A = -\frac{7}{2}}$$

$$a \cdot b = -30 \quad (*)$$

$$2 \text{ \& } 4 \rightarrow A = 3$$



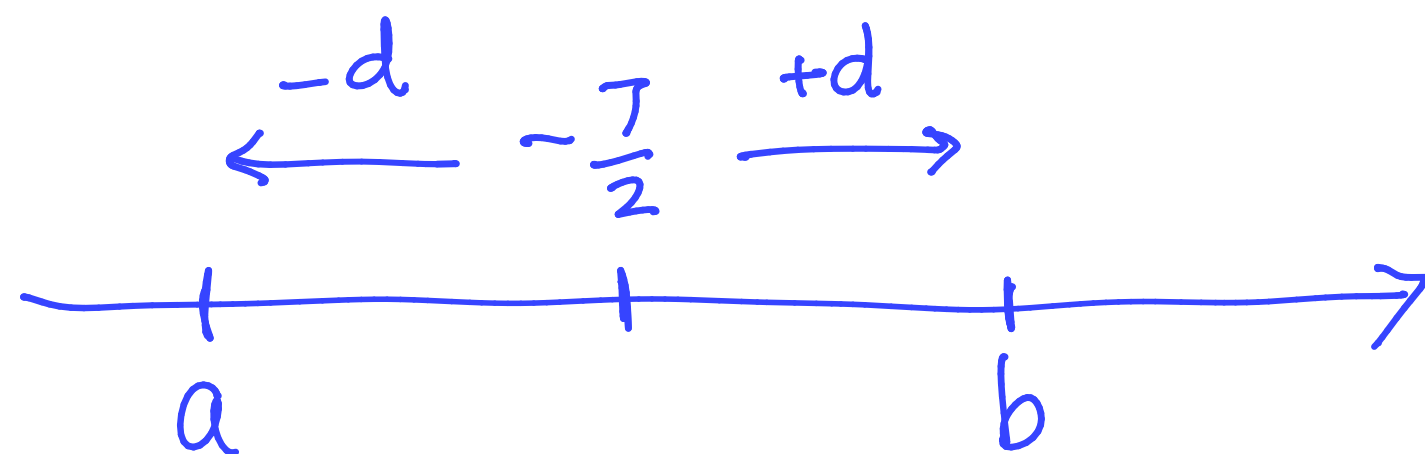
$$a = A - d \quad \& \quad b = A + d$$

$$(*) \quad ab = (A - d)(A + d) = A^2 - d^2 = -30$$

$$\rightarrow \left(-\frac{7}{2}\right)^2 - d^2 = -30 \rightarrow \frac{(-7)^2}{2^2} - d^2 = -30 \rightarrow \frac{49}{4} - d^2 = -30$$

$$\frac{49}{4} = d^2 - 30 \rightarrow \frac{49}{4} + \frac{120}{4} = d^2 \rightarrow \frac{169}{4} = d^2$$

$$\sqrt{\frac{169}{4}} = \sqrt{d^2} \rightarrow \frac{13}{2} = d$$



$$-\frac{7}{2} + \frac{13}{2} = \frac{+6}{2} = 3 \quad \& \quad -\frac{7}{2} - \frac{13}{2} = \frac{-7-13}{2} = \frac{-20}{2} = -10$$

$$(x + a)(x + b) = (x + 3)(x - 10)$$

$$\underline{\text{Ex}} : w^2 - 62w + 432 = (x + a)(x + b)$$

$$A = -31$$

$$(A + d)(A - d) = 432 \rightarrow A^2 - d^2 = 432 \rightarrow 961 - d^2 = 432$$

$$d^2 = 961 - 432 \rightarrow d^2 = 529 \rightarrow d = 23$$

$$\left. \begin{array}{l} a = A + d = -31 + 23 \rightarrow a = -8 \\ b = A - d = -31 - 23 \rightarrow b = -54 \end{array} \right\} (x - 8)(x - 54)$$

$$\text{Ex: } \overset{A}{6}y^2 - \overset{B}{13}y + \overset{C}{6} = 0$$

$$\downarrow$$

$$\underline{6y^2 - 9y - 4y + 6}$$

$$3y(2y-3) - 2(2y-3)$$

$$(2y-3)(3y-2)$$

$$\left. \begin{array}{l} AC = 36 \text{ prod} \\ B = -13 \text{ sum} \end{array} \right\} -9 \& -4$$

$$6y^2 - 13y + 6 = 0$$

$$(2y-3)(3y-2) = 0$$

$$\underset{+3}{2y} - \underset{+3}{3} = 0 \quad \text{or} \quad \underset{+2}{3y} - \underset{+2}{2} = 0$$

$$\frac{2y}{2} = \frac{3}{2}$$

$$\boxed{y = \frac{3}{2}}$$

$$\frac{3y}{3} = \frac{2}{3}$$

$$\boxed{y = \frac{2}{3}}$$

$$\sqrt{100} = 10, \sqrt{16} = 4, \sqrt{28} = ?$$

$$\rightarrow \sqrt{4} = 2, \sqrt{9} = 3, \sqrt{25} = 5, \sqrt{49} = 7, \sqrt{121} = 11.$$

$$\sqrt{28} = \sqrt{4 \cdot 7} = \sqrt{4} \cdot \sqrt{7} = 2\sqrt{7}.$$

$$\text{Ex: Simplify } \sqrt{50}.$$

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

$$\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2}.$$

$$\text{Ex: Simplify } \sqrt{45}.$$

$$\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Quadratic formula

$$\underset{\substack{\downarrow \\ a}}{1}x^2 - \underset{\substack{\downarrow \\ b}}{6}x - \underset{\substack{\downarrow \\ c}}{3} = 0 \rightarrow \text{Prime polynomial} \quad AC = -3$$

$$B = -6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$= \frac{6 \pm \sqrt{36 + 12}}{2}$$

$$= \frac{6 \pm \sqrt{48}}{2}$$

$$= \frac{6 \pm 4\sqrt{3}}{2}$$

$$\rightarrow \frac{6}{2} \pm \frac{4\sqrt{3}}{2} = 3 \pm 2\sqrt{3}$$

$$\rightarrow \frac{\cancel{2}(3 \pm 2\sqrt{3})}{\cancel{2}} = 3 \pm 2\sqrt{3}$$

$$\text{Solution: } x = 3 + 2\sqrt{3} \text{ or } x = 3 - 2\sqrt{3}$$

$$\text{Ex: Solve } \overset{a}{6}x^2 + \overset{b}{10}x - \overset{c}{5} = 0 \text{ for } x.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(10) \pm \sqrt{10^2 - 4 \cdot 6 \cdot (-5)}}{2 \cdot 6}$$

$$= \frac{-10 \pm \sqrt{100 + 120}}{12} = \frac{-10 \pm \sqrt{220}}{12}$$

$$= \frac{-10 \pm 2\sqrt{55}}{12} = \frac{\cancel{2}(-5 \pm \sqrt{55})}{\cancel{12} 6} = \frac{-5 \pm \sqrt{55}}{6}$$

$$x^2 = 6x + 3$$

$$x^2 - 6x - 3 = 0$$

$$\sqrt{48} = \sqrt{4 \cdot 12} = \sqrt{4} \cdot \sqrt{12}$$

$$= 2\sqrt{12}$$

$$\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4} \cdot \sqrt{3}$$

$$= 2\sqrt{3}$$

$$\sqrt{48} = 2 \cdot \sqrt{12} = 2 \cdot (2\sqrt{3})$$

$$= 4\sqrt{3}$$

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

$$\left\{ \begin{aligned} \sqrt{220} &= \sqrt{4 \cdot 55} \\ &= \sqrt{4} \cdot \sqrt{55} \\ &= 2\sqrt{55} \end{aligned} \right.$$

$$\Delta = b^2 - 4ac$$

discriminant

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

$$\Delta \begin{cases} \Delta > 0: 2 \text{ real answers} \\ \Delta = 0: 1 \text{ real answer} \\ \Delta < 0: 2 \text{ complex answers} \end{cases}$$

Ex: $x^2 + \underline{2}x + 4 = 0$. How many solutions are there?

$$\Delta = 2^2 - 4 \cdot 1 \cdot 4 \rightarrow \Delta = 4 - 16 \rightarrow \Delta = -12 \quad (2 \text{ cplx solns})$$

$$ax^2 + bx + c = 0.$$

$$\begin{aligned} \sqrt{50} &= \sqrt{25 \cdot 2} \\ &= \sqrt{25} \cdot \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

$$\underline{\text{Ex}}: \quad \begin{array}{ccc} x^2 - 50 = 0. \\ \quad \quad \quad +50 \quad \quad +50 \end{array}$$

$$x^2 = 50 \rightarrow \cancel{\sqrt{x^2}} = \pm \sqrt{50} \rightarrow x = \pm 5\sqrt{2}$$

$$\underline{\text{Ex}}: \quad \begin{array}{ccc} 3x^2 - 147 = 0. \\ \quad \quad \quad +147 \quad \quad +147 \end{array}$$

$$\frac{\cancel{3}x^2}{\cancel{3}} = \frac{147}{3} \rightarrow x^2 = 49 \rightarrow \sqrt{x^2} = \pm \sqrt{49}$$

$$\rightarrow x = \pm 7.$$